



UNIVERSITI PUTRA MALAYSIA

**FLUTTER ANALYSIS OF A SCALED MODEL OF AN EAGLE 150B/AC
WING**

AZMIN SHAKRINE BIN MOHD RAFIE

FK 2007 73

**FLUTTER ANALYSIS OF A SCALED MODEL OF AN EAGLE 150B/AC
WING**

By

AZMIN SHAKRINE BIN MOHD RAFIE

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfilment of the Requirements for the Degree of Doctor of Philosophy**

September 2007



DEDICATION

Alhamdulillah thanks to Allah the Almighty for the blessings and opportunities that He has provided the strength for me to accomplish this PhD study. This thesis is especially dedicated to;

My father and mother:

Mohd Rafie Hussain and Mariatul Kibtiah Mohd Yatim, thank you for your support, understanding and blessing to carry out this study.

My beloved wife:

Norazlinda Mohd Darby who constantly source of inspiration, motivation, encouragement and support throughout this study.

My son:

Muhammad Zunnurain, source of my inspiration.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment
of the requirement for the degree of Doctor of Philosophy

**FLUTTER ANALYSIS OF A SCALED MODEL OF AN EAGLE 150B/AC
WING**

By

AZMIN SHAKRINE BIN MOHD RAFIE

September 2007

Chairman: Professor Ir. ShahNor Basri, PhD

Faculty: Engineering

An investigation of the problem of the flutter condition of an Eagle 150B aircraft wing is undertaken. The research is largely devoted to investigating the adequacy of the ideal flutter theory that has been employed to predict flutter boundary for such wing. A series of panel flutter experiment carried out in UPM 1m × 1m wind tunnel at Mach number up to 0.132 are described in detail. Furthermore, an extensive parametric computational analysis has been conducted to improve flutter condition by reconfiguring the wing design specification. For experimental analysis, the ground test which includes static and dynamic tests of the wind model has been performed followed by the wind tunnel testing. The data gathered from the wind tunnel testing is analyzed using the logarithmic decrement method so that the flutter speed can be predicted. The wing model mounting system test rig has been designed and developed together with the data acquisition system which is used for data collection. In order to validate the experimental technique, wind tunnel testing using three different types of materials for rectangular flat plate has been conducted. The types of materials used are aluminum 6061, mild steel and stainless steel. The

agreement between experimental technique and computational analysis is acceptable since the error of difference is less than 6 percent.

MSC. Patran and Nastran software have been used to predict the flutter condition since it has the capability to carry out the aeroelasticity analysis of the actual wing and wing model. The PK-method and aerodynamic doublet lattice methods were selected for this analysis as it provides the eigenvalue solutions in the form of the V-g and V-f graphs. Validation of the computational analysis with two existing published results is performed to ensure the results are reliable. The parametric study produced the results on the effects of the mass, altitude, span length, stiffness and center of gravity position against the flutter speed condition. This research work may conclude that both techniques are reliable to investigate flutter speed since the validation results showed a good agreement. It was also found that through extensive parametric study, several suggestions have been made to reconfigure the wing in order to improve the flutter condition.

Abstrak tesis dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**ANALISIS FLUTER UNTUK MODEL BERSKALA BAGI SAYAP
PESAWAT EAGLE 150B**

Oleh

AZMIN SHAKRINE BIN MOHD RAFIE

September 2007

Pengerusi: Profesor Ir. ShahNor Basri, PhD

Fakulti: Kejuruteraan

Penyiasatan bagi masalah keadaan fluter untuk sayap pesawat Eagle 150B telah dilaksanakan. Penyelidikan ini menumpukan sepenuhnya untuk menyiasat kecekapan teori unggul fluter yang telah digunakan untuk meramal sempadan fluter sayap tersebut. Beberapa siri eksperimen panel fluter yang dijalankan di terowong angin $1\text{m} \times 1\text{m}$ UPM pada nombor Mach sehingga 0.132 telah dinyatakan dengan terperinci. Selanjutnya, kajian parameter yang mendalam menggunakan kaedah analisis berkomputer telah dilakukan untuk memperbaiki lagi keadaan fluter dengan mengubah spesifikasi rekabentuk sayap. Untuk analisis eksperimen, ujian di dataran yang mengandungi ujian statik dan dinamik untuk model sayap telah dilaksanakan diikuti oleh ujian terowong angin. Data yang diperolehi daripada ujian terowong angin akan dianalisa menggunakan kaedah pengurangan logaritma di mana kelajuan fluter akan dapat diramal. Sistem rig ujian pencagak model sayap telah direkabentuk dan dibangunkan bersama dengan sistem perolehan data yang digunakan untuk mengumpul data. Untuk tujuan pengesahan analisis eksperimen, ujian terowong angin dengan menggunakan tiga jenis bahan yang berbeza untuk plat rata segi empat tepat telah dilakukan. Jenis bahan yang digunakan adalah aluminium 6061, besi

lembut dan keluli tahan karat. Kesesuaian antara teknik eksperimen dan analisis berkomputer adalah diterima memandangkan perbezaan ralat adalah kurang daripada 6 peratus.

Perisian MSC. Patran dan Nastran telah digunakan untuk meramal keadaan flutter di mana ia mempunyai kebolehan untuk menjalankan analisis keanjalan udara untuk analisis sayap sebenar dan sayap model. Kaedah PK dan kaedah aerodinamik gandaan kekisi telah dipilih untuk analisis ini di mana ia memberikan penyelesaian nilai eigen di dalam bentuk graf V-g dan V-f. Pengesahan analisis berkomputer dengan dua keputusan penerbitan yang sedia ada telah dilakukan untuk memastikan keputusan yang boleh dipercayai. Kajian parameter memberikan hasil kesan jisim, ketinggian, panjang sayap, keanjalan dan kedudukan pusat gravity terhadap keadaan kelajuan flutter. Kerja penyelidikan ini dapat menyimpulkan bahawa kedua teknik berkebolehan untuk menyiasat keadaan flutter disebabkan keputusan pengesahan menunjukkan kesesuaian yang baik. Melalui kajian parameter yang mendalam, beberapa cadangan juga dapat dirumuskan untuk mengubah sayap pesawat dengan tujuan untuk memperbaiki lagi keadaan flutter.

ACKNOWLEDEMENTS

First and foremost, Alhamdulillah thanks to Allah the Almighty for the blessings and opportunities that He has provided the strength for me to accomplish this PhD study. I would like to express my sincere gratitude to my advisor, Professor Dr. Ir. ShahNor Basri, who constantly source out good advice, encouragement and all kind of support throughout this study. He has consistently put the interest of his students first and always kept an open door policy. I am deeply indebted and most grateful to Dr.-Ing. Ir. Renuganth Varatharajoo and Dr. Ir. Prasetyo Edi, the supervisory committee members, who have guided this work and for their helpfulness discussion and advice. I'm highly appreciated for their time and willingness to serve as my supervisory committee.

I wish to extend my thanks to the staff of the Department of Aerospace Engineering for their friendly dealing and moral support. I'm gratefully acknowledged the help and patience of Mr. Ropiee Mat and Mr. Ahmad Saiful Abu Samah, the technicians of the Aerodynamic Lab and Structure Lab. I also thank my friends and colleagues especially tutor form Aerospace Department for sharing their knowledge and encouragement. Thanks must also go to my family members, my wife and son for their patience, understanding and encouragement as well as the wonderful environment they provided for the successful completion of this work. And lastly, I gratefully acknowledge the support of the Composites Technology Research Malaysia (CTRM) for the information and data of Eagle 150B aircraft wing and allow me to study this type of aircraft.

APPROVAL

I certify that an Examination Committee met on 14 September, 2007 to conduct the final examination of Azmin Shakrine Bin Mohd Rafie on his degree of Doctor of Philosophy thesis entitled “Flutter Analysis of A Scaled Model of an Eagle 150B/AC Wing” in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulation 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

Megat Mohamad Hamdan Megat Ahmad, PhD

Associate Professor
Faculty of Engineering
Universiti Putra Malaysia
(Chairman)

Faizal Mustapha, PhD

Faculty of Engineering
Universiti Putra Malaysia
(Internal Examiner)

Rizal Zahari, PhD

Faculty of Engineering
Universiti Putra Malaysia
(Internal Examiner)

Nik Abdullah Nik Mohammad, PhD

Professor
Faculty of Engineering
Universiti Kebangsaan Malaysia
(External Examiner)

HASANAH MOHD GHAZALI, PhD

Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 24 October 2007



This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy.

Shahnor Basri, PhD

Professor
Faculty of Engineering
Universiti Putra Malaysia
(Chairman)

Renuganth Varatharajoo, PhD

Lecturer
Faculty of Engineering
Universiti Putra Malaysia
(Member)

Prasetyo Edi, PhD

Lecturer
Faculty of Engineering
Universiti Putra Malaysia
(Member)

AINI IDERIS, PhD

Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 15 November 2007



DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any degree at UPM or other institutions.

AZMIN SHAKRINE BIN MOHD RAFIE

Date: 28 April 2008

TABLE OF CONTENTS

	Page
DEDICATION	ii
ABSTRACT	iii
ABSTRAK	v
ACKNOWLEDGEMENTS	vii
APPROVAL	viii
DECLARATION	x
LIST OF TABLES	xiv
LIST OF FIGURES	xv
LIST OF ABBREVIATIONS	xx

CHAPTER

1 INTRODUCTION

1.1	Introduction	1
1.2	Objectives and scopes of research	4
1.3	Thesis Layout	5

2 LITERATURE REVIEW

2.1	Aeroelasticity	7
2.2	Flutter	9
2.3	Experimental Aeroelasticity	12
2.3.1	Early Development	12
2.3.2	Present Study	17
2.3.3	Experimental Setup	21
2.4	Computational Aeroelasticity	26
2.4.1	Early Development	26
2.4.2	Present Study	28
2.4.3	Computational Study	32
2.5	Closure	38

3 THEORY

3.1	Structural Dynamics of Typical Wing Section	40
3.2	Aerodynamics	42
3.2.1	Unsteady Aerodynamic	42
3.2.2	Analytical Solution Method	46
3.2.3	Integral Equation Method	47
3.2.4	Numerical Solution Method	49
3.3	Flutter Solution Methods	50
3.3.1	Flutter Solution for Typical Section	50

3.3.2	The K-Method	53
3.3.3	The PK-Method	56
3.4	Closure	57
4	EXPERIMENTAL MODEL DEVELOPMENT	
4.1	Wing Model Calculations	58
4.1.1	Wing Bending Calculation	59
4.1.2	Wing Torsion Calculation	60
4.1.3	Wing Model Similarity Laws Calculation	61
4.2	Wing Model Design	62
4.2.1	Material Properties Testing	63
4.2.2	Structural Stiffness of Wing Model	68
4.2.3	Design of Wing Model	72
4.2.4	Mass Balance of Wing Model	74
4.3	Wing Model Fabrications and Assembly	79
4.4	Closure	84
5	EXPERIMENTAL SETUP	
5.1	Aeroelasticity Experiment	85
5.1.1	Ground Test	86
5.1.2	Wind Tunnel Setup	94
5.1.3	Wind Tunnel Test	114
5.2	Closure	116
6	COMPUTATIONAL FLUTTER ANALYSIS	
6.1	Computational Modeling	118
6.1.1	Structural Modeling	119
6.1.2	Aerodynamic Modeling	124
6.1.3	Structural and Aerodynamic Interaction	126
6.1.4	Flutter Analysis	127
6.2	Computational Procedures	128
6.3	Computational Analysis Validation	130
6.4	Closure	133
7	RESULTS AND DISCUSSION	
7.1	Experimental Aeroelasticity Analysis Results	135
7.2	Computational Analysis Results	145
7.2.1	Convergence Analysis	145
7.2.2	Computational Analysis Result for Actual Wing	148
7.2.3	Computational Analysis Result for Wing Model	151
7.3	Parametric Study	154
7.3.1	Mass Effect	154
7.3.2	Air Density Effect	157
7.3.3	Stiffness Effect	159
7.3.4	Wing Span Effect	162

7.3.5	Center of Gravity Effect	164
7.4	Closure	166
8	CONCLUSION AND RECOMMENDATIONS	
8.1	Conclusion	168
8.2	Recommendations for Future Work	171
8.3	Contributions of Work	172
	REFERENCES	174
	APPENDICES	183
	BIODATA OF THE AUTHOR	226
	LIST OF PUBLICATIONS	227



LIST OF TABLES

Table	Page
4.1 Unidirectional Carbon Fiber/Epoxy Properties	68
4.2 Reduce Stiffness Coefficients	71
4.3 Locations of Mass for Each Station of Unweighted Wing Model from LE	76
4.4 Mass Moment of Inertia for Unweighted Wing Model at CG_d	77
5.1 Bending Stiffness Corresponding to Average Deflection	88
5.2 Torsional Stiffness Corresponding to Average Angle Deflection	91
5.3 Normal Modes Results for Eleven Different Positions of Accelerometer	93
5.4 Normal Modes Results Comparison between Actual Wing and Wing Model	94
5.5 Digital Manometer and Tube Manometer Reading for Corresponding Hz	102
5.6 Digital manometer and Tube Manometer Reading for Corresponding Velocity	103
5.7 Dynamic Pressure (Pa) at Different Locations of YZ-Plane	106
5.8 Results for Wind Tunnel Testing and Computational Analysis	113
6.1 Validation Results for Present Computational Analysis Technique	132
8.1 Flutter Speed Prediction using Experimental and Computational Analysis	170



LIST OF FIGURES

Figure	Page
1.1 Catastrophic Flutter Effects to Electra Aircraft Model	2
1.2 General View of Eagle 150B aircraft	4
2.1 The Aeroelasticity Triangle of Forces	8
2.2 Wing bending and torsion coupling	10
2.3 Induced angle of attack due to bending oscillations	11
2.4 Wind Tunnel Mounting System	22
2.5 Spars and Pod Construction	25
2.6 Features of Transonic Flutter	33
2.7 Coupled Fluid Structural Analysis	36
2.8 Varying Levels of Fidelity in Modeling for Fluids and Structures	37
3.1 Typical Wing Section	41
3.2 Oscillating Airfoil	43
3.3 Flow around Thick Cambered Airfoil	44
3.4 Pressure Distribution Acting along Airfoil	45
3.5 Plot of the Real and Imaginary Parts of Theodorsen's Function $C(k)$	47
3.6 Circulation along airfoil and wake	47
3.7 Plot of $\omega_{1,2}/\omega_0$ and g versus $U/(b\omega_0)$ using k method	55
3.8 Comparison between p-k and k methods of Flutter Analysis	57
4.1 Wing Modeled as Uniform Cantilever Beams	59
4.2 Wing Modeled as Simple Symmetrical Shapes and Lumped Masses	60
4.3 Tensile Test Specimens	64



4.4	Tensile Stress (σ_1) versus Tensile Strain (ϵ_1)	64
4.5	Tensile Stress (σ_2) versus Tensile Strain (ϵ_2)	65
4.6	Schematic of a $[\pm 45]$ Laminated Tensile Coupon	66
4.7	Tensile Stress (τ_{12}) versus Tensile Strain (γ_{12})	67
4.8	Coordinate System for the Composite Material	69
4.9	Diagram for Angle of the Lamina (θ)	70
4.10	Detailed Design of the Rectangular Wing Spar Model	72
4.11	Airfoil Shape for Scale Eagle 150B Aircraft Wing	73
4.12	Plan View of the Wing Model	73
4.13	Separated Rigid Sections with Balsa Wood	74
4.14	Locations of Center of Gravity for Additional Mass (m_Δ)	77
4.15	Locations for Split Additional Mass m_f and m_r	78
4.16	Unidirectional Carbon/Epoxy Spar for Wing Model	80
4.17	Ribs with Hollow Aluminum Tube	80
4.18	Skin Fabrication Attached Between the Ribs	81
4.19	Spar and Ribs Assembly with Leading Edge and Trailing Edge Supports	82
4.20	Additional Mass Added	82
4.21	Strain Gauge Added	83
4.22	Complete Assembly of Wing Model for Wind Tunnel Testing	84
5.1	Dial Gauge Attached at Wing Spar Model Tip Loaded with Weights	87
5.2	Results from Static Bending Testing for Five Reading	87
5.3	Protractor Attached at Wing Spar Model Tip Loaded with Weights	89
5.4	Results from Static Torsional Testing for Five Readings	90

5.5	Eleven Different Locations of the Accelerometer during Dynamic Testing	92
5.6	Main Frame	96
5.7	Rotating Disk and Clamp System	97
5.8	Brake System with Support System	97
5.9	Complete Sidewall Mounting System and Location in the Wind Tunnel	98
5.10	Low Speed Wind Tunnel at Universiti Putra Malaysia	100
5.11	Graf of Velocity from Digital and Tube Manometer versus Motor RPM	104
5.12	Schematic View of Wind Tunnel Test Section	105
5.13	Dynamic Pressure Variations from the Mean (%) at Test Section	107
5.14	Hole Location at Wind Tunnel Test Section	108
5.15	Airflow Velocity Profile for Wind Tunnel Test Section	109
5.16	Flat Plate Geometry of Aluminum 6061, Mild Steel and Stainless Steel	109
5.17	Sensor location	110
5.18	Experimental Results for Flat Plate Validation Test	111
5.19	Computational Results for Flat Plate Validation Test	113
5.20	Wing Model Mounted Inside the Wind Tunnel	112
6.1	Locations of the Forward Mass (m_f) and Rear Mass (m_r) from EA	120
6.2	One-dimensional Beam Element Structural Modeling for Actual Wing	120
6.3	Locations of Center of Gravity (CG) for Additional Mass (m_Δ)	122
6.4	Locations of the Forward Mass ($m_{\Delta f}$) and Rear Mass ($m_{\Delta r}$) from EA	123

6.5	Two-dimensional Shell Element Structural Modeling for Wing Model	124
6.6	Aerodynamic Modeling for Actual Wing	126
6.7	Structural and Aerodynamic modeling Grids Connections for Actual Wing	127
6.8	Computational Procedure Flow Chart for Structural Analysis	128
6.9	Computational Procedures Flow Chart for Aeroelasticity Analysis	129
6.10	The Computational Analysis Results for Square Plat Model	131
6.11	The Computational Analysis Results for 15 Degree Swept Wing Model	132
7.1	Bending and Torsion Deflection for 0 m/s Wind Tunnel Airflow Speed (m/s)	137
7.2	Bending Deflection for every Increments of Wind Tunnel Speed (m/s)	140
7.3	Enlarged Graph for Bending Deflection Data at 0 m/s Condition	142
7.4	Wing Model Damping Constant against Wind Tunnel Airflow Speed	143
7.5	Destruction of the Wing Model at 40 m/s	145
7.6	Five Different Numbers of Elements for Finite Element Structural Modeling	146
7.7	Normal Modes Frequencies Results for Wing Model	147
7.8	Normal Modes Frequencies Results for Actual Wing	148
7.9	Computational Analysis Results for Actual Wing	149
7.10	The V-g and V-f plots for Actual Wing at Flutter Mode (Mode 3)	151
7.11	Computational Analysis Results for Wing Model	152
7.12	The V-g and V-f plots for Wing Model at Flutter Mode (Mode 6)	153

7.13	Computational Results for Different Mass Parameter at Flutter Mode	155
7.14	Graph of Flutter Speed against Different Mass Parameter	156
7.15	Computational Results for Different Altitude at Flutter Mode	158
7.16	Graph of Flutter Speed against Different Flight Altitude	158
7.17	Computational Results for Different Stiffness Factor at Flutter Mode	160
7.18	Graph of Flutter Speed against Different Stiffness Factor	161
7.19	Computational Results for Different Wing Span at Flutter Mode	163
7.20	Graph of Flutter Speed against Different Wing Span Length	163
7.21	Computational Results for Different CG Locations at Flutter Mode	165
7.22	Graph of Flutter Speed against Variable CG Distance to EA	166

LIST OF ABBREVIATIONS

AGARD	Advisory Group for Aerospace Research and Development
AR	Aspect Ratio
ARES	Aeroelasticity Rotor Experimental System
ASTM	Standard Test Method
CAE	Computational Aeroelasticity
CAP-TSD	Computational Aeroelasticity Program – Transonic Small Disturbance
CFD	Computational Fluid Dynamic
CG	Center of Gravity
CSM	Computational Structural Mechanics
CTRM	Composites Technology Research Malaysia
DAQ	Data Acquisition
DLM	Doublet-Lattice Method
EA	Elastic Axis
EE	Euler equations
FE	Finite Element
FFT	Fast Fourier Transformation
FP	Full Potential
Hz	Hertz
ICW	Intermediate Complexity Wing
LCO	Limit Cycle Oscillation
LE	Leading Edge
MSC Nastran	MacNeal-Schwendler Corporation – NASA Structural Analysis

NASA	National Aeronautics and Space Administration
PAPA	Pitch and Plunge Apparatus
RAE	Royal Aeronautical Establishment
RBAR	Rigid Bar
RPM	Rotations per Minute
SDOF	Single Degree of Freedom
TDT	Transonic Dynamics Tunnel
TE	Trailing Edge
TLNS	Thin Layer Navier-Stokes Equations
TSD	Transonic Small Disturbance
TWG	Transonic Wind Tunnel Gottingen
VS	Virtual Surface

Nomenclature

$[A(t)]$	Matrix of unsteady aerodynamics operators
A	Constant
c	Chord
C	Cos (θ)
$C(k)$	Theodorsen's function
C_p	Pressure data
$[D]$	Damping matrix
e	Exponential
E	Modulus of elasticity
E_1	Longitudinal Young's modulus
E_2	Transverse Young's modulus

EI	Bending stiffness
f_f	Flutter frequency
g	Damping factor
G	Shear modulus of elasticity
G_{12}	In-plane shear modulus
GJ	Torsional stiffness
$H_n^{(2)}(k)$	Hankel functions of the second kind
h, α	Deflection in bending and torsional directions
\bar{i}, \bar{k}	A unit of Cartesian vector
i	imaginary number
I	Moment of inertia
I_α	Mass moment of inertia
J	Polar moment of inertia
$[K]$	Stiffness matrix
k_t	Constant thickness for each layers
k	Reduced frequency
K	Kernel function
K_α	Torsional stiffness
K_h	Bending stiffness
l, L	Span length
L	Vertical force per unit span
$[M]$	Mass matrix
m	Mass per unit length
M_∞	Mach number
M_y	Moment force about y axis per unit span

N_{lay}	Total number of layers
P_t	Load at tip
P	Pressure
P_∞	Pressure at free stream
$\{q\}$	Vector of generalized coordinates
\bar{Q}	Transformed reduced stiffness coefficients
q	Dynamic pressure
Q	Reduced stiffness coefficients
q_1, q_2	Generalized coordinate
Q_h, Q_a	Generalized forces
q_i	ith generalized coordinate
R	Gas constant for air
\vec{r}	Displacement of any point
r	Radius of gyration
S	Sin (θ)
S_α	Static moment
T°	Atmospheric temperature
T	Kinetic energy
t	Time
T_t	Torsional force at tip
T_d	Damped period
u	Horizontal displacement
U	Potential energy
V, V_∞	Free stream velocity
V_b	Vertical velocity vector due to bending

V_f	Flutter speed
V_r	Resulting velocity vector at wing
w	Vertical displacement
w_a	Vertical velocity component over airfoil contour
x	Deflection in x direction
X_1, X_2	Amplitudes
X_{cg}	Distance from center of gravity
$\{Z\}$	Force vector
Z	Distance from center line
z_a	Vertical displacement of airfoil at point x, z and t

Greek Symbols

α	Geometric angle of attack
ε_1	Longitudinal tensile strain
σ_2	Transverse tensile stress
α_{eff}	Effective angle of attack of wing
α_{ind}	Induced angle of attack due to bending
θ_t	Angle deflection at tip
γ_a, γ_w	Circulation along airfoil and wake
δ	Logarithmic decrement
δ_t	Deflection at tip
ε_2	Transverse tensile strain
ζ	Damping ratio
θ	Angle of lamina
μ	Mass ratio